# Lecture 7: Span of vectors part 1

September 29, 2016 10:08 PM

We've seen 2 ways of describing subspaces:

(1) $\mathrm{w} = \{\text{things} \mid \text{conditions on things}\}$

eg. $\mathrm{w} = \{(x, y, z) \in \mathbb{R} \mid x - 2y + z = 0\}$

(2) $\mathrm{u} = \{\text{things with parameters} \mid \text{parameters are real}\}$

eg. $u = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Big| \, a, b, c \in \mathbb{R}\}$

$u = \left\{ a \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Big| \, a, b, c \in \mathbb{R} \right\}$

"take all linear combinations of these elements"

## 6.1 Converting from (1) to (2)

$W = \{(x, y, z) \in \mathbb{R}^3 \mid x - 2y + z = 0\}$
$= \{(x, y, z) \mid z = -x + 2y\}$
$= \{(x, y, -x + 2y) \mid x, y \in \mathbb{R}\}$
$= \{x * (1,0,-1) + y * (0,1,2) \mid x, y \in \mathbb{R}\}$

## 6.2 Definition of a span

Let V be a vector space, and $\{v_1, v_2, \dots, v_m\} \subseteq \mathrm{V}$, a set of vectors in V.

(1) If $a_1, a_{2,\dots}, a_m \in \mathbb{R}$, then $a_1 v_1 + a_2 v_2 + \cdots + a_m v_m$ is called a linear combination of $v_1, v_{2,\dots}, v_m$

(2) span $\{v_1, v_2, \dots, v_m\}$
$= \{a_1 v_1 + a_2 v_2 + \cdots + a_m v_m \mid a \in \mathbb{R}\}$
$=$ set of all linear combinations of $v_1, v_{2,\dots}, v_m$

(3) A vector space W is spanned by $v_1, \dots, v_m$ if $\mathrm{W} = \mathrm{span}\{v_n, \dots, v_m\}$

Example

$s\mathrm{pan}\{(1,0), (0,1)\} = \mathbb{R}^2$

$s\mathrm{pan}\underbrace{\{(1,2)}, (3,6)\} \neq \mathbb{R}^2$

$\{(x, 2x) \mid x \in \mathbb{R}\}$

## 6.3 The big theorem

Let V be a vector space, $\{v_1, \dots, v_m\} \subseteq \mathrm{V}$, and let $\mathrm{U} = \mathrm{span}\{v_1, \dots, v_m\}$.

i) U is a subspace of V

ii) if W is a subspace of V with $\{v_1, \dots, v_m\} \subseteq \mathrm{W}$ then $\mathrm{U} \subseteq \mathrm{W}$

therefore U is the smallest subspace containing $\{v_1, \dots, v_m\}$

**PROOF:**

i) apply the subspace test

(1) $0 = 0 * v_1 + \cdots + 0 * v_m \in \mathrm{U}$

(2) $u = a_1 v_1 + \cdots + a_m \cdot v_m, \mathrm{v} = b_1 v_1 + \cdots + b_m v_m$.
Then, $u + v = (a_1 + b_1) * v_1 + \cdots + (a_m + b_m) * v_m \in \mathrm{U}$

(3) $c \in \mathbb{R}, u = a_1 v_1 + \cdots + a_m v_m$. Then, $cu = (ca_1) * v_1 + \cdots + (ca_m) * v_m \in \mathrm{U}$

Example

$W = \{(x, y, z - y) \mid x, y \in \mathbb{R}\}$
$= \{x * (1,0,1) + y * (0,1 - 1) \mid x, y \in \mathbb{R}\}$
$= \mathrm{span}\{(1,0,1), (0,1,-1)\}$
Therefore, subspace of $\mathbb{R}^3$.